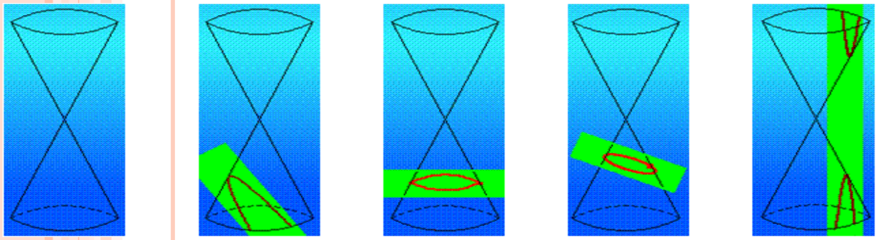

Quadratic Relations and Conic Sections

Algebra 2

Chapter 9

Algebra II 9

- This Slideshow was developed to accompany the textbook
  - *Larson Algebra 2*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

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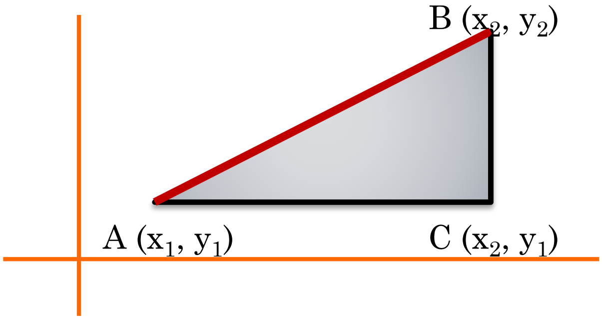
9.1 Apply the Distance and Midpoint Formulas

- Distance Formula
  - $d^2 = AC^2 + BC^2$
  - $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$



- $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

9.1 Apply the Distance and Midpoint Formulas

- Find the distance between(1, -3) and (2, 5)
- What type of triangle is ΔRST if R(2, -2), S(4, 2), T(6, 0)?

$d = \sqrt{((-2-1)^2 + (5-(-3))^2)} = \sqrt{73} = 8.54$

RS = $\sqrt{20}$
ST = $\sqrt{8}$
RT = $\sqrt{20}$
Isosceles (remind students that the other choices are scalene and equilateral)

9.1 Apply the Distance and Midpoint Formulas

- Midpoint formula
- $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

- Find the midpoint of (1, -3) and (-2, 5)

((1+-2)/2, (-3+5)/2) = (-1/2, 1)

9.1 Apply the Distance and Midpoint Formulas

- Find the equation of a perpendicular bisector
  1. Find the midpoint
  2. Find the slope
  3. Write the equation of the line using the midpoint and the negative reciprocal of the slope

9.1 Apply the Distance and Midpoint Formulas

- Find the perpendicular bisector of segment AB if A(-2, 1) and B(1, 4).

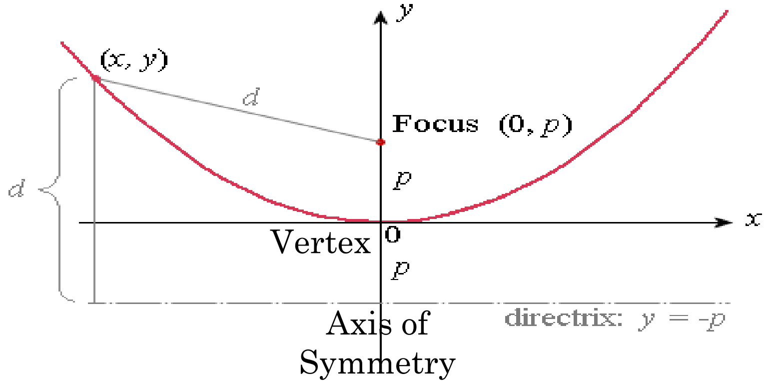
Midpoint: ((-2+1)/2, (1+4)/2) = (-1/2, 5/2)
Slope: (4-1)/(1-(-2)) = 3/3 = 1
Equation: $y - y_1 = m(x - x_1)$
y – 5/2 = -1(x – (-1/2)) → y – 5/2 = -x – ½ → y = -x + 2

Quiz

- 9.1 Homework Quiz

9.2 Graph and Write Equations of Parabolas

- Parabola
  - Shape of the graph of a quadratic equation
  - All the points so that the distance to the focus and to the directrix is equal



9.2 Graph and Write Equations of Parabolas

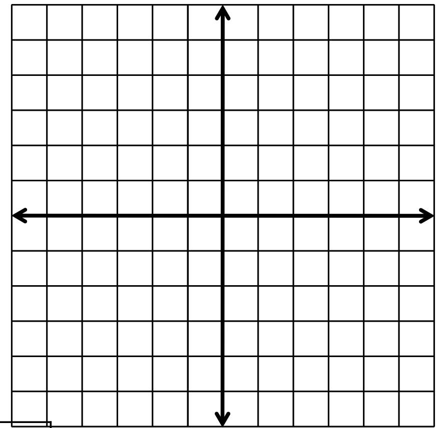
- Standard Equation of a Parabola (vertex at origin)

Equation	Focus	Directrix	Axis	Opens
$x^2 = 4py$	(0, p)	y = -p	x = 0	up
$y^2 = 4px$	(p, 0)	x = -p	y = 0	right

If p is negative, the parabola opens the other direction

9.2 Graph and Write Equations of Parabolas

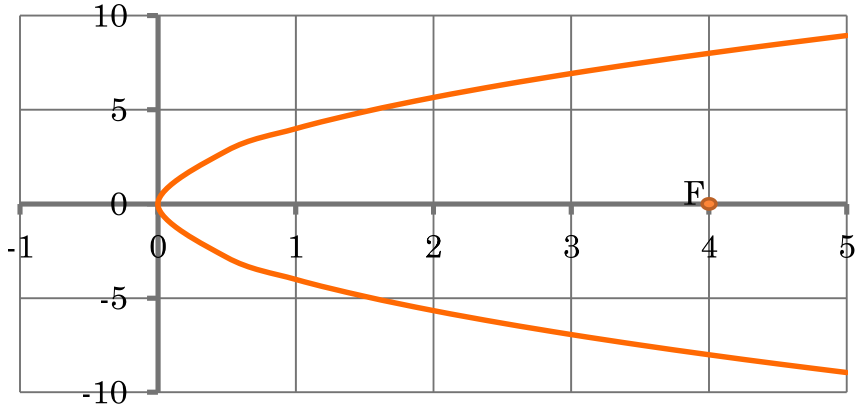
- Identify the focus, directrix, and graph $x = 1/8\ y^2$
  - Solve for squared term
    $y^2 = 8\ x$
  - Coefficient of non-squared term = 4p
    8 = 4p
    p = 2
  - Plot the directrix and focus
    x = -2, (2, 0)
  - Plot other points from a table of values



x	y
2	-4, 4
1	$-2\sqrt{2}, 2\sqrt{2}$

9.2 Graph and Write Equations of Parabolas

- Write the equation for the parabola.



p = 4

$y^2 = 4(4)x \rightarrow y^2 = 16x$

Quiz

- 9.2 Homework Quiz

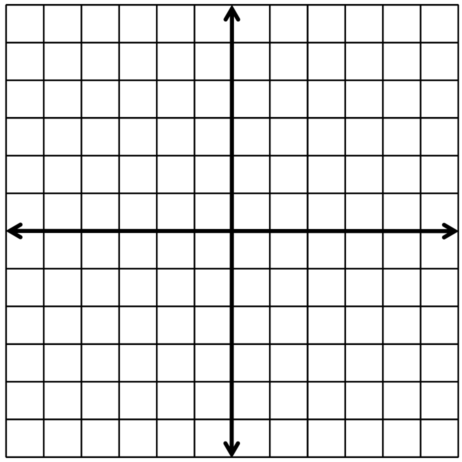
9.3 Graph and Write Equations of Circles

- Circle
  - Set of points a fixed distance (radius) from the center

- Derivation of equation (center at origin)
  - r = distance from center
  - $r = \sqrt{(x-0)^2 + (y-0)^2}$
  - $r^2 = x^2 + y^2$
  - $x^2 + y^2 = r^2$

9.3 Graph and Write Equations of Circles

- To graph
  - Find the radius
  - Plot the center (0, 0)
  - Move up, down, left, and right from the center the distance of the radius
  - Draw a good circle
- Graph $x^2 + y^2 = 16$



r = 4

9.3 Graph and Write Equations of Circles

- Write the equation of a circle with center at the origin and goes through point (-3, 5)

$x^2 + y^2 = r^2$
$(-3)^2 + 5^2 = r^2$
$9 + 25 = r^2$
$34 = r^2$
$x^2 + y^2 = 34$

9.3 Graph and Write Equations of Circles

- Finding a tangent line to a circle
  - Tangent lines are perpendicular to the radius
  - Find the slope of the radius to the point of intersection
  - Use the negative reciprocal of the slope as the slope of the tangent line
  - Use the slope and the point of intersection to write the equation of the line

9.3 Graph and Write Equations of Circles

- Find the equation of the tangent line at (1, 5) to $x^2 + y^2 = 26$

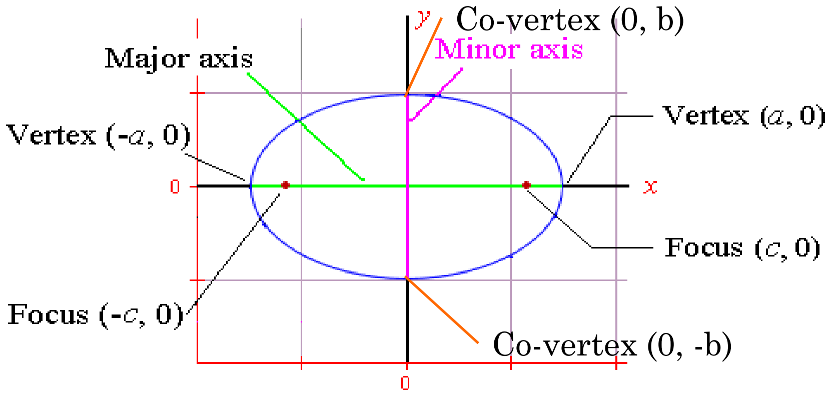
$m_r = (5-0)/(1-0) = 5$
$m_{tan} = -1/5$
$y - 5 = -1/5(x - 1) \rightarrow y - 5 = -1/5x + 1/5 \rightarrow y = -1/5x + 26/5$

QUIZ

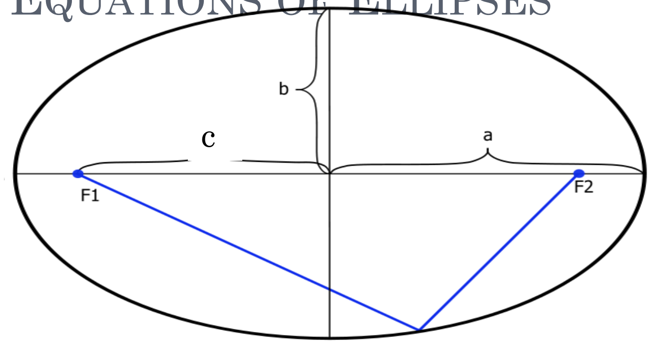
- 9.3 Homework Quiz

9.4 Graph and Write Equations of Ellipses

- Set of points so that the sum of the distances to the 2 foci is constant



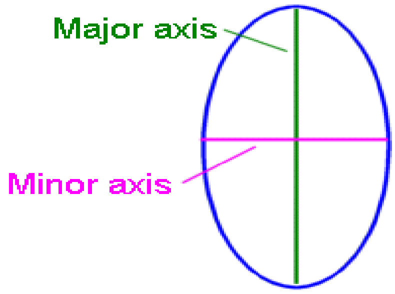
9.4 Graph and Write Equations of Ellipses



- Horizontal Ellipse.
  - Center at origin
  - $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
    - $a > b$
    - $c^2 = a^2 - b^2$

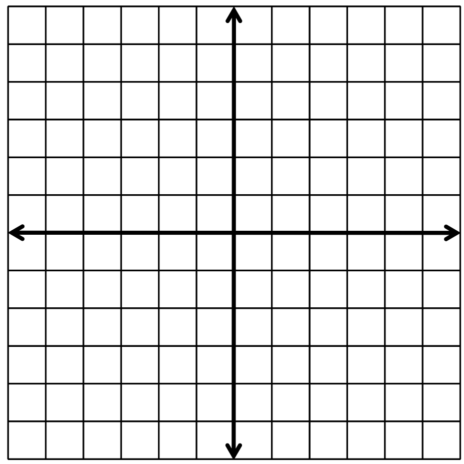
9.4 Graph and Write Equations of Ellipses

- Vertical Ellipse.
  - Center at origin
  - $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
    - $a > b$
    - $c^2 = a^2 - b^2$



9.4 Graph and Write Equations of Ellipses

- Graph Ellipse
  - Write in standard form (find a and b)
  - Plot vertices and co-vertices
  - Draw ellipse
- Graph $4x^2 + 25y^2 = 100$ and find foci



horizontal
$x^2 / 25 + y^2 / 4 = 1$
$a = 5; b = 2$

Foci: $c^2 = a^2 – b^2$
$c^2 = 25 – 4 = 21$
$c = √21$
$(-√21, 0), (√21, 0)$

9.4 Graph and Write Equations of Ellipses

- Write the equation for an ellipse with center at (0, 0) and …
  - a vertex at (0, 5), and a co-vertex at (4, 0)

a = 5, b = 4, y is bigger than x so y-axis is major axis

$x^2/16 + y^2/25 = 1$

9.4 Graph and Write Equations of Ellipses

- Write the equation for an ellipse with center at (0, 0) and …
  - A vertex at (-6, 0) and a focus at (3, 0)

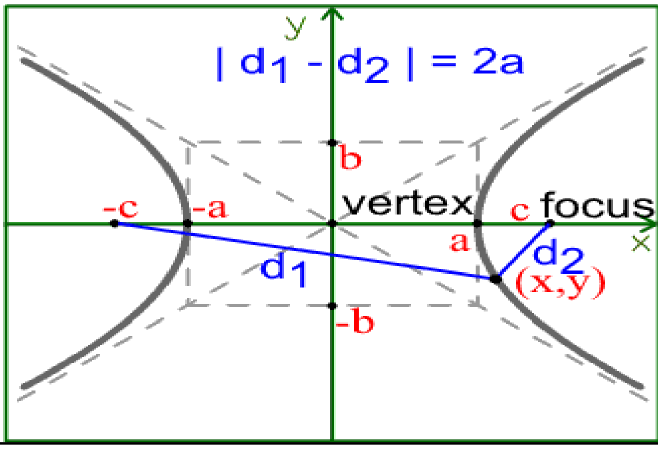
$a = 6$, $c = 3$,
$c^2 = a^2 - b^2 \rightarrow 9 = 36 - b^2 \rightarrow b^2 = 27$
X is major axis
$x^2/36 + y^2/27 = 1$

Quiz

- 9.4 Homework Quiz

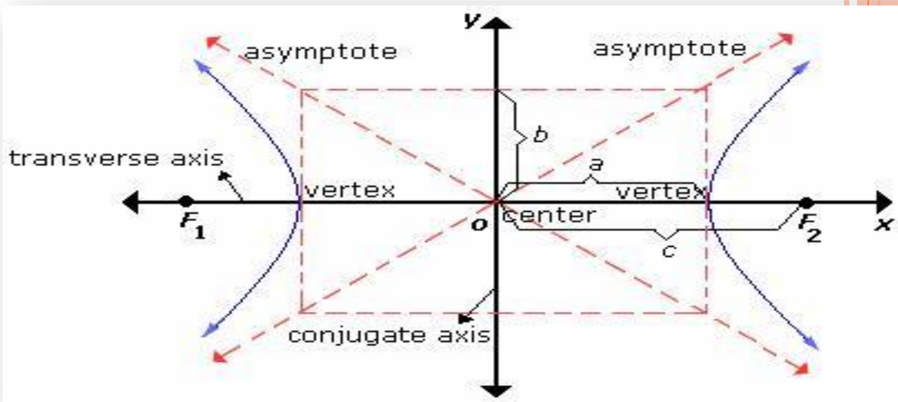
9.5 Graph and Write Equations of Hyperbolas

- Set of all points so the difference of the distances between a point and the two foci is constant



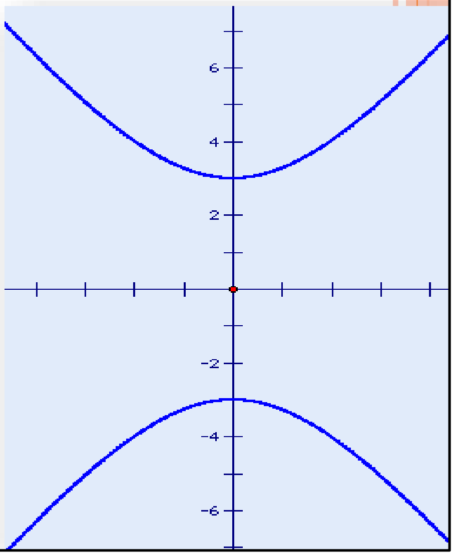
9.5 Graph and Write Equations of Hyperbolas

- Horizontal transverse axis
- $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
- $c^2 = a^2 + b^2$
- Asymptotes
- $y = \pm \frac{b}{a} x$



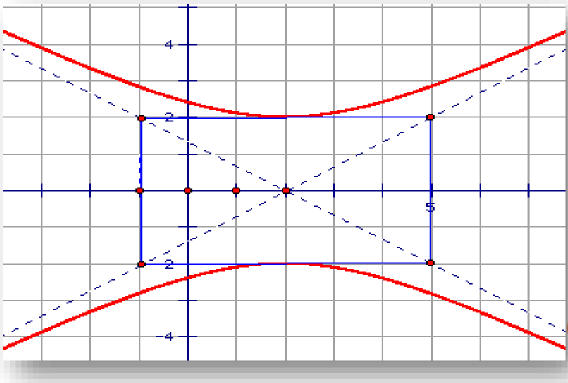
9.5 Graph and Write Equations of Hyperbolas

- Vertical transverse axis
- $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$
- $c^2 = a^2 + b^2$
- Asymptotes
- $y = \pm \dfrac{a}{b} x$



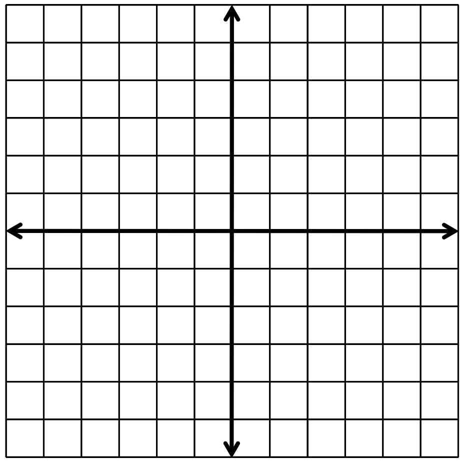
9.5 Graph and Write Equations of Hyperbolas

- Graphing Hyperbolas
  - Plot the vertices and "co-vertices"
  - Draw the "box"
  - Draw the asymptotes
  - Draw the hyperbola



9.5 Graph and Write Equations of Hyperbolas

- Graph $9x^2 - 16y^2 = 144$



Rewrite $9x^2/144 - 16y^2/144 = 1 \rightarrow x^2/16 - y^2/9 = 1$
a = 4, b = 3, x is first so horizontal

9.5 Graph and Write Equations of Hyperbolas

- Write the equation of hyperbola with foci (0, -5) and (0, 5) and vertices at (0, -3) and (0, 3).

vertical
$c = 5$, $a = 3$
$c^2 = a^2 + b^2 \rightarrow 25 = 9 + b^2 \rightarrow b^2 = 16$
$y^2/9 - x^2/16 = 1$

Quiz

- 9.5 Homework Quiz

9.6 Translate and Classify Conic Sections

- Remember when we studied quadratics and absolute value equations?

- $y = a(x - h)^2 + k$

- h is how far the graph moved right
- k is how far the graph moved up

- We can apply this concept for conics, too.

9.6 Translate and Classify Conic Sections

- Circle: $(x-h)^2 + (y-k)^2 = r^2$

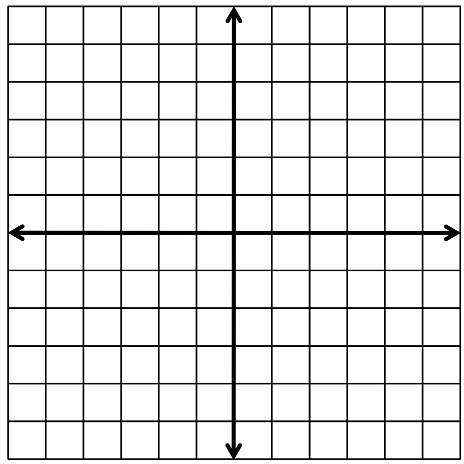
	Horizontal Axis	Vertical Axis
Parabola:	$(y-k)^2 = 4p(x-h)$	$(x-h)^2 = 4p(y-k)$
Ellipse:	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
Hyperbola:	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

9.6 Translate and Classify Conic Sections

- How to graph
  - Find the center/vertex (h, k)
  - Graph the rest as before

9.6 Translate and Classify Conic Sections

- Graph
- $(x+1)^2 + (y-3)^2 = 4$



- $(x+3)^2 - \frac{(y-4)^2}{4} = 1$

Circle: center (-1, 3), radius = 2

Hyperbola: center (-3, 4), a = 1, b = 2

9.6 Translate and Classify Conic Sections

- Write equations of a translated conic
  - Graph known points to determine horizontal or vertical axis
  - Find the center/vertex to give (h, k)
  - Use the known points to find a and b (or p)

9.6 Translate and Classify Conic Sections

- Write an equation of a parabola with vertex (3, -1) and focus at (3, 2).

- Write an equation of a hyperbola with vertices (-7, 3) and (-1, 3) and foci (-9, 3) and (1, 3).

Parabola: h = 3, k = -1; p = distance from focus to vertex = 3.
Vertical axis: $(x-h)^2 = 4p(y-k)$ → $(x-3)^2 = 4(3)(y+1)$ → $(x-3)^2 = 12(x+1)$

Hyperbola: Horizontal axis
Center midpoint between vertices: $\left(\frac{-1+(-7)}{2}, \frac{3+3}{2}\right) \rightarrow (-4, 3)$
a = distance from center to vertex = 3
c = distance from center to focus = 5
$c^2 = a^2 + b^2$ → $5^2 = 3^2 + b^2$ → $16 = b^2$ → b = 4

$$\frac{(x+4)^2}{9} - \frac{(y-3)^2}{16} = 1$$

9.6 Translate and Classify Conic Sections

- Identify lines of symmetry
- Conics are symmetric along their axes which go through their center/vertex

- $\frac{(x-5)^2}{64} + \frac{y^2}{16} = 1$

- $(x + 5)^2 = 8(y - 2)$

Ellipse: center (5, 0) → lines of symmetry: x = 5; y = 0

Parabola: vertex (-5, 2) → vertical axis: line of symmetry: x = -5

9.6 Translate and Classify Conic Sections

- Classifying Conics from general equations
- $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

- Discriminant: $B^2 - 4AC$

- $B^2 - 4AC < 0$, $B = 0$ and $A = C$ Circle
- $B^2 - 4AC < 0$, $B \neq 0$ or $A \neq C$ Ellipse
- $B^2 - 4AC = 0$ Parabola
- $B^2 - 4AC > 0$ Hyperbola

- If $B = 0$, the axes are horizontal or vertical.
- If $B \neq 0$, the axes are rotated

9.6 Translate and Classify Conic Sections

- An asteroid's path is modeled by $4x^2 + 6.25y^2 - 12x - 16 = 0$ where x and y are in astronomical units from the sun. Classify the path and write its equation in standard form.

A = 4, B = 0, C = 6.25

$B^2 - 4AC = 0^2 - 4(4)(6.25) = -100$ → ellipse

Complete the square in x and y to get in standard form.

$4x^2 - 12x + 6.25y^2 = 16$

$4(x^2 - 12x + ?) + 6.25y^2 = 16 + 4(?)$

$4(x^2 - 3x + (3/2)^2) + 6.25y^2 = 16 + 4(2.25)$

$4(x - 1.5)^2 + 6.25y^2 = 25$

$$\frac{4(x-1.5)^2}{25} + \frac{6.25y^2}{25} = 1$$

$$\frac{(x-1.5)^2}{6.25} + \frac{y^2}{4} = 1$$

Quiz

- 9.6 Homework Quiz

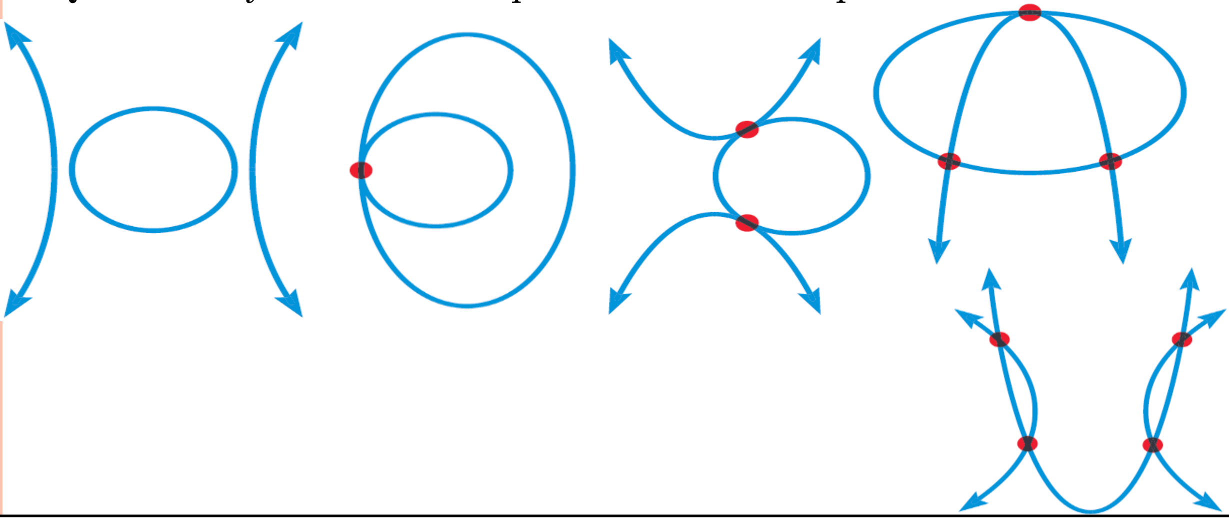
9.7 Solve Quadratic Systems

- You have already learned how to solve systems using
  - Graphing
  - Substitution
  - Elimination

- You can use all three methods to solve quadratic systems.

9.7 Solve Quadratic Systems

- Quadratic systems of two equations can have up to four solutions.



9.7 Solve Quadratic Systems

- Solve using substitution
  - $y^2 - 2x - 10 = 0$
    $y = -x - 1$

$y^2 - 2x - 10 = 0$
$y = -x - 1$

$(-x-1)^2 - 2x - 10 = 0$
$x^2 + 2x + 1 - 2x - 10 = 0$
$x^2 - 9 = 0$
$x^2 = 9$
$x = \pm 3$

$y = -(3) - 1 = -4$
$y = -(-3) - 1 = 2$

9.7 Solve Quadratic Systems

- Solve using elimination
  - $$x^2 + 4y^2 + 4x + 8y = 8$$
    $$y^2 - \quad x + 2y = 5$$

$$x^2 + 4y^2 + 4x + 8y = 8$$
$$y^2 - \quad x + 2y = 5$$

$$x^2 + 4y^2 + 4x + 8y = 8$$
$$-4y^2 + 4x - 8y = -20$$

$$x^2 + 8x = -12$$
$$x^2 + 8x + 12 = 0$$
$$(x + 6)(x + 2) = 0$$
$$x = -2, -6$$

$$y^2 - x + 2y = 5$$
$$y^2 + 2y - (x + 5) = 0$$
$x = -2$: $y^2 + 2y - (-2 + 5) = 0$

$(y - 1)(y + 3) = 0$

$y = 1, -3$

Points are (-2, 1), (-2, -3)

$x = -6$: $y^2 + 2y - (-6 + 5) = 0$

$(y + 1)^2 = 0$

$y = -1$

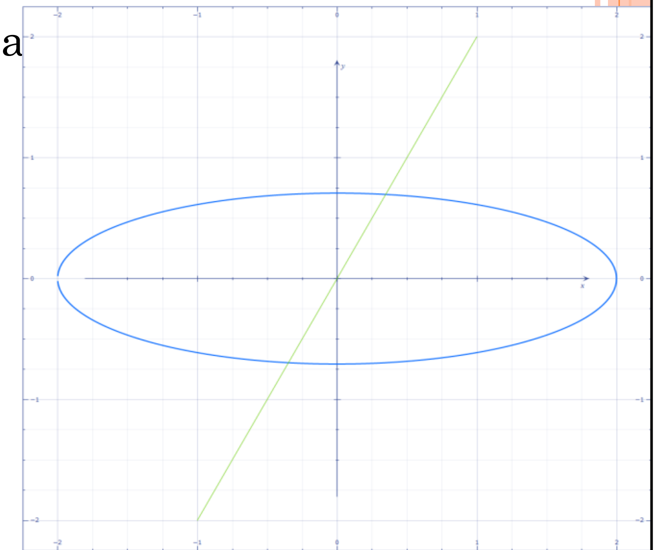
Points are (-6, -1)

9.7 Solve Quadratic Systems

- Solve by graphing calculator
  - Graph both equations
    - You will have to solve for y.
    - If you have a ± sign, then you will have to graph one equation for the + and one for the --
  - On TI-83/84
    - Push $\boxed{2nd}$ $\boxed{CALC}$
    - Choose "intersect"
    - Push enter for the first curve
    - Push enter for the second curve (you may have to use the up/down arrows to choose the right curve)
    - Use the left and right arrows to move the cursor to an intersection and push enter.
    - Repeat for the rest of the intersections

9.7 Solve Quadratic Systems

- Solve using a graphing calcula
  - $x^2 + 8y^2 - 4 = 0$
    $y = 2x$



Points are (.34815531, .69631062), (-.34815531, -.69631062)

Quiz

- 9.7 Homework Quiz